Limits of precision measurements based on interferometers

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ABSTRACT

Laser interferometric methods are employed in precision measurements and positioning tasks, since they provide the means for attaining high metric resolution and precision, even over large measurement ranges. The most important fundamental principles of heterodyne and homodyne interferometers are discussed. A metrological analysis makes it possible to describe the advantages and limits of laser interferometry. The design and functionality of fibre-coupled miniature interferometers are described. The broad applicability of interferometers to microtechnology, nanotechnology and precision mechatronics is explained.

Keywords: Heterodyne and homodyne interferometers, metrological analysis, nanometrology, nano-precision devices

1. INTRODUCTION

Today's nanometrology limits the accuracy of precision engineering. These limits are based on the metre definition as redefined in 1983 as well as on the comparison between an iodine-stabilised helium-neon laser and a stabilised helium-neon laser to be calibrated, on the influence of the refractive index of the air, on the realisation of the Abbe principle and so on. Laser interferometric devices are employed in precision measurements and positioning tasks, since they provide a means for attaining high metric resolution and precision, even over long measurement ranges. The versatility and broad applicability of laser interferometers are unattainable using any other metrological methods. Laser interferometers suitable for use in advanced areas of mecrotechnology, nanotechnology and precision mechatronics must have small measuring heads that are insensitive to environmental effects. Miniature interferometers fulfilling this requirement have been developed at the Institute of Process Measurement and Sensor Technology of the Technische Universität Ilmenau and are manufactured by SIOS Messtechnik GmbH. Their metrological benefits are due to their compact dimensions and their utilisation of optical fibres for coupling the laser light source, measuring head and electronic signal processing unit.

The most important fundamental principles of heterodyne and homodyne interferometric devices are discussed. Their benefits and limitations are covered based on a metrological analysis and the broad applicability is explained by different precision mechatronic devices.

2. FUNDAMENTALS OF INTERFEROMETERS

2.1 The "Interference" in Interferometers

The following section covers only interferometers with amplitude division. It is sensible to differentiate between interferometers with neutral beam splitters and ones with polarising beam splitters.

Homodyne interferometers usually use neutral beam splitters, whereas heterodyne devices require polarising elements.

The dispersion of light is described by the Maxwellian wave equations. For homogenous isotropic non-conductors it holds [1, 2]:

$$\Delta\vec{E} - \frac{1}{v^2}\ddot{\vec{E}} = 0 \qquad (1)$$

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$$\Delta\vec{H} - \frac{1}{v^2}\ddot{\vec{H}} = 0 \qquad (2)$$

where $\vec{E}$ is the electrical field strength vector, $\vec{H}$ the magnetic field strength vector, ν the phase velocity of the wave in a medium with the refractive index n where $\nu = c_0/n$ and c_0 the phase velocity of the wave in a vacuum.

If no birefringent medium is present, the dispersion of the light can be described by equation (1). A solution to the equation is possible with the assumption of monochromatic light:

$$\vec{E}(\vec{r},t) = \vec{a}(\vec{r}) \cdot e^{j\left[\vec{k}\cdot\vec{r} - \varpi t + \vartheta\right]} \qquad (3)$$

$\vec{a}(\vec{r})$ is the vector amplitude, $\vec{k} = \frac{2\pi}{\lambda} \cdot \vec{e}_k$ is the wave number vector, $\vec{r}$ the position vector and ϑ the phase for t = r = 0.

Figure 1 shows the simplified set-up of a heterodyne interferometer. In order to calculate the intensity distribution I, it is assumed that the polarising beam splitter (PBS) feeds beam ω_1 into one interferometer arm and ω_2 into the other arm. Furthermore, it is presumed that two planar, monochromatic, linearly polarised waves with frequencies ω_1 and ω_2 interfere.

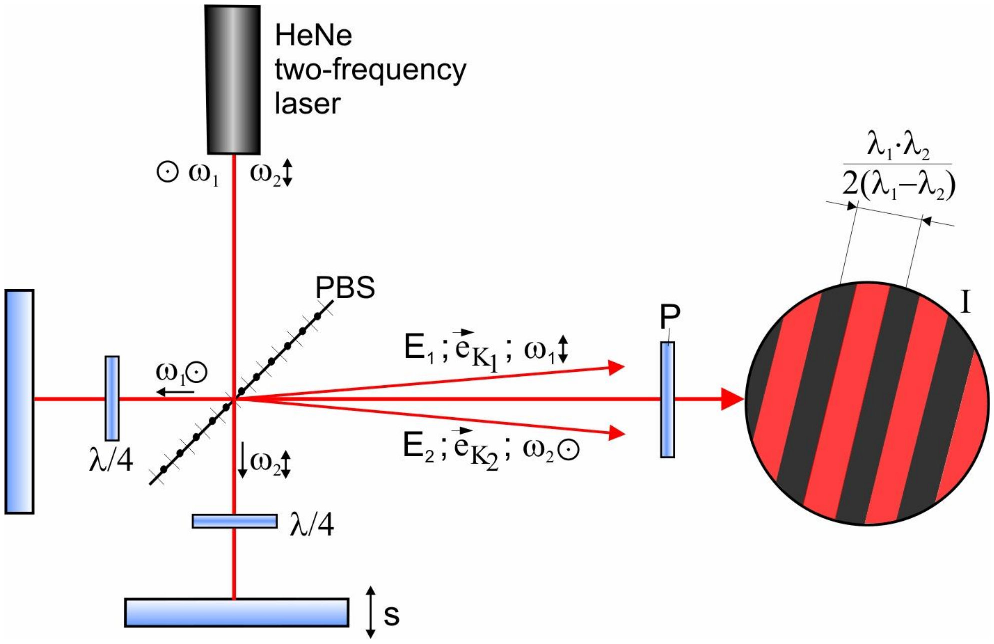

Fig. 1: Heterodyne interferometer

The following relationships are used to calculate the intensity distribution:

$$I = \frac{\varepsilon \cdot \varepsilon_0 \cdot c}{2} \cdot E \cdot E^* = \frac{\varepsilon \cdot \varepsilon_0 \cdot c}{2} a^2 \qquad (4)$$

where E is a complex value, E^* its complex conjugate value, a the amplitude.

From

$$E = \sum_i E_i \qquad (5)$$

with

$$E_1\left(\vec{r},t\right) = a_1 \cdot e^{j\left[\vec{r}\vec{k}_1 - \omega_1 t + \vartheta_1\right] = j\gamma_1}$$

$$E_2\left(\vec{r},t\right) = a_2 \cdot e^{j\left[\vec{r}\vec{k}_2 - \omega_2 t + \vartheta\right] = j\gamma_2}$$

and

$$2\cos\left(\gamma_1 - \gamma_2\right) = e^{j(\gamma_1 - \gamma_2)} + e^{-j(\gamma_1 - \gamma_2)}$$

it follows that

$$I_p = I_{1_p} + I_{2_p} + 2\sqrt{I_{1_p} I_{2_p}} \cos\left(\gamma_1 - \gamma_2\right) \tag{6}$$

(the subscript p denotes polarised)

$$\gamma_1 - \gamma_2 = \gamma = \vec{r}\left(\vec{k}_1 - \vec{k}_2\right) + \left(\omega_2 - \omega_1\right)t + \vartheta_1 - \vartheta_2 \tag{7}$$

The interference orders change with the frequency $\omega_2 - \omega_1$, yielding a synthetic wavelength $\lambda_{syn} = \frac{\lambda_1 \cdot \lambda_2}{\lambda_1 - \lambda_2}$.

In contrast to heterodyne interferometers, homodyne interferometers operate using one frequency. Therefore, it holds for $\omega_1 = \omega_2$:

$$\gamma = \vec{r}\left(\vec{k}_1 - \vec{k}_2\right) + \vartheta_1 - \vartheta_2 \tag{8}$$

The phase γ only changes with a change in the optical path difference.

In the following section only homodyne interferometers will be discussed.

If an interferometer reflector is displaced a distance s, it follows that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\gamma + \gamma_M\right) \tag{9}$$

and

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\gamma + \frac{2\pi}{\lambda_0} \cdot n \cdot i \cdot s\right) \tag{10}$$

γ describes the interference pattern before the reflector displacement and γ_M contains the phase change resulting from the movement of the reflector by s.

i is termed interferometer factor; the interferometer in fig. 1 possesses a factor i = 2. From equation (10) it follows for the displacement s:

$$s = \frac{k \cdot \lambda_0}{i \cdot n} = \frac{k \cdot c_0}{i \cdot n \cdot f_{HeNe_{stab}}} \tag{11}$$

where λ_0 is the wavelength of light in vacuum, k the change of the interference order with a displacement s of the reflector and $f_{HeNestab}$ the frequency of a stabilised He-Ne laser.

2.2 Metrological analyses

2.2.1 Resolution

Next, the smallest resolvable displacement s_q must be determined. If an interference order (k = 1) is divided into e electronic increments, it follows from equation (11) that:

$$s_q = \frac{\Delta k \cdot \lambda_0}{i \cdot n} = \frac{\lambda_0}{e \cdot i \cdot n} \tag{12}$$

with $\Delta k = \frac{1}{e}$, where Δk is the smallest resolvable interference order and e is the number of electronic increments per interference order. Given that $\frac{\lambda_0}{n} = 632.8\,nm$, i = 2 and e = 256, s_q is then determined to be only 1.24 nm.

2.2.2 Uncertainty of measurement

Some parameters affecting the uncertainty of measurement of s can be derived from equation (11).

c_0:

The big advantage of the metre definition of 1983 is that the value of the speed of light in a vacuum was fixed at a value of 299 792 458 m/s.

k:

The number of impulses N counted during the displacement of a reflector by s can be calculated by equation 13:

$$N = \frac{i \cdot e \cdot n}{\lambda_0} \cdot s \tag{13}$$

Nonlinearities of the interpolation electronics must be considered when determining N.

n:

With the help of the Edlen formula, the dependence of the refractive index of the air on air pressure, humidity and temperature can be compensated for within certain limits.

The following relative uncertainty results from the empirically determined Edlen formular as well as from the measurement uncertainty of the sensors:

$$\frac{\Delta n}{n} = 5 \cdot 10^{-8}$$

This means that measurements over long ranges (> 30 mm) must take place in a vacuum, with a vacuum of 10^{-3} bar being sufficient.

$f_{HeNestab}$:

The frequency $f_{HeNestab}$ must be known as indicated in equation (11). For this, the difference is determined between $f_{HeNestab}$ and the frequency of an iodine-stabilised He-Ne laser [3, 4], the frequency of which is coupled to the caesium frequency standard f_{cs133}. The wavelength of the iodine-stabilised He-Ne laser resulting from the traceable connection is λ_{HeNe} = 632.99139822 nm with a relative standard uncertainty of $2.5 \cdot 10^{-11}$ [4] (see fig. 2).

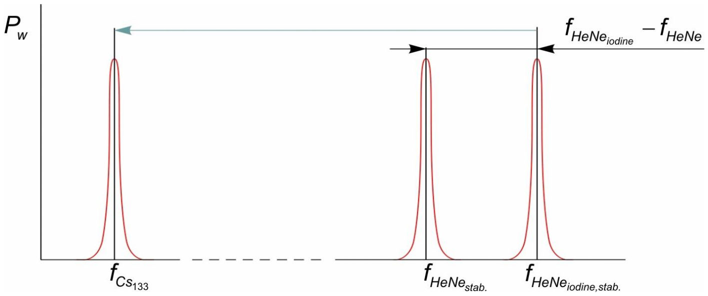

Fig. 2: Traceability of a stabilised He-Ne laser

Further influences affecting the measurement uncertainty of interferometric measurements must also be minded. Such influences can be caused for example by not observing the Abbe comparator principle or as a result of the instability of the metrological frame.

3. APPLICATIONS OF FIBRE-COUPLED MINIATURE INTERFEROMETERS IN PRECISION MECHATRONIC DEVICES

Figure 3 depicts a block schematic of a fibre-coupled miniature interferometer system.

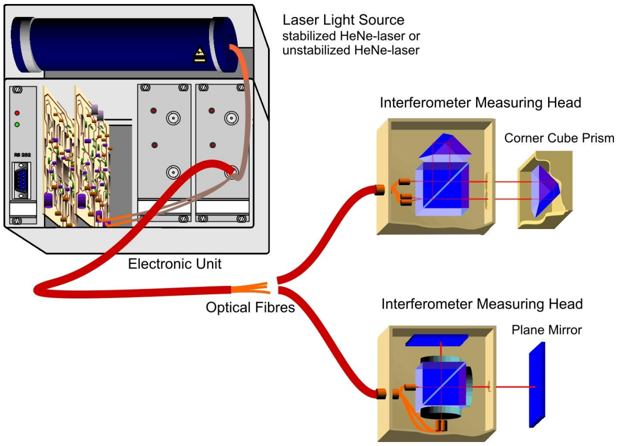

Fig. 3: Block schematic of a fibre-coupled miniature interferometer system

Both types of interferometers – i.e., those equipped with plane mirrors and those equipped with corner cube reflectors – are needed for applications in nanopositioning and nanomeasuring in the fields of microtechnology, nanotechnology and precision mechatronics. Only plane-mirror interferometers allow the taking of three-dimensional coordinate measurements [5, 6]. Interferometers equipped with corner cube reflectors are relatively insensitive to large reflector angular misalignment errors [7].

In the last few years, fibre-coupled miniature interferometers have been used to develop innovative precision mechatronic devices. Metrological atomic force microscopes, laser vibrometers, interference-optical force sensors and nanoindenters as well as nanometre-precision positioning stages are some of the many examples of the high performance of miniature interferometers [8].

A metrological scanning force microscope is described here as an example of such a technology [8, 9, 10]. In order to yield a metrological scanning force microscope whose measurement results may be traced to international standards, a miniature plane-mirror interferometer is incorporated into the microscope such that compliance with the Abbe comparator principle is maintained over all measurement areas (see Fig. 4). Our conversion of a “VERITEKT” scanning force microscope to a metrological one was accomplished under subcontract and working in collaboration with the PTB in Germany. An improvement in expanded uncertainty of 0.2 nm (k = 2) along a structural standard was obtained.

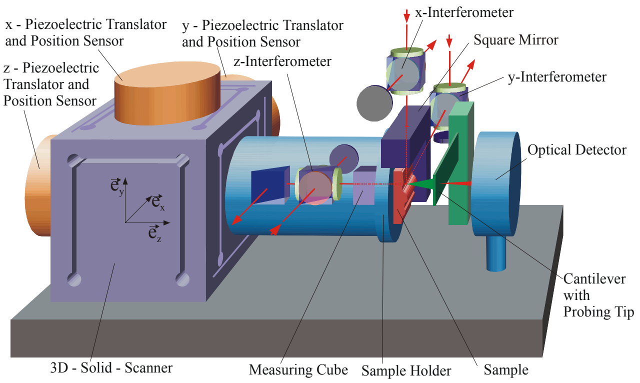

Fig. 4: A metrological scanning force microscope

Another example is the laser interferometric force sensor [11]. The gravitational force F acting on a weight deflects the parallel-spring arrangement, whose parallel springs are fabricated from top-quality quartz glass. Detection of their deflection due to the gravitational force acting on a weight employs an all-fibre-optic-coupled miniature interferometer equipped with a corner-cube retroreflector. In this particular case, employment of a corner-cube retroreflector turns out to be beneficial, since its insensitivity to angular misalignments eliminates effects on interference patterns due to angular displacements and out-of-plane distortions of the parallel springs. This laser interferometric force sensor allows the attaining of a resolution of 10 µN over a dynamic range of 0…0.1 N and improves the expanded uncertainty of measurement to better than 10 µN. Resolutions better than 1 µN are feasible. The force sensor described here has been successfully employed for testing (calibrating) applanation tonometers.

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